conceptual overview!

Direct Proof best for $P \Rightarrow Q$ style statements

- assume P, follow logical deduction to get to Q

Contraposition best for $P \Rightarrow Q$ style statements

- proof $\neg Q \Rightarrow \neg P$ by direct proof (so assume $\neg Q$)
- sometimes Q will have more properties than P to use to start your proof

Contradiction best for P style statements

- assume $\neg P$, prove that things go Horribly Wrong™
  - usually that two contradicting things are <u>both</u> true
- might need to assume something more too
  - if working with rational / irrational, usually that $k = a/b$, assume $a, b \in \mathbb{Z}$ + share no factors
- if working with implication, convert $P \Rightarrow Q$ to $\neg P \vee Q$
  - $\neg(\neg P \vee Q) = P \wedge \neg Q$, so essentially trying to "prove" that P <u>can</u> exist without Q (... but can it? :o)

Proof by Cases

- ensure that all your cases encompass all the possibilities
- best way to build intuition for this one is by doing examples

Proof by Induction

- Sometimes just gotta try a few approaches
- can use lemmas gone over in class in your proofs
- DO NOT ASSUME THE STATEMENT YOU'RE TRYING TO PROVE

... i am watching

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conceptual overview!

Remember...

- don't assume the whole statement you're trying to prove
- don't feel like a proof is "better" if it's more "mathy"
  - → goal is to be precise and concise
  - → if both not possible, shoot for precise
- DO try different approaches to build intuition on which to use
- if want to prove $\forall x \in$ universe, $P(x)$ *:
  take an arbitrary element $\in$ universe and prove $P(x)$ → now its true $\forall x$
- * Sometimes it's hard to directly prove $\forall x$, in that case, try contradiction (what happens if $\exists x, \neg P$?)
- if this is difficult now, don't worry! intuition comes with practice

key takeaways

① you CAN try contradiction on implications, but be extra careful to negate it correctly (recommend $\neg P \vee Q$ format)

② cool math facts

- $a \in \mathbb{Z}$, $2 \mid a \Rightarrow \exists k \in \mathbb{Z}$ st $a = 2k$ (a is even)
  $2 \nmid a \Rightarrow \exists k \in \mathbb{Z}$ st $a = 2k+1$ (a is odd)
  } parity

- $k \in \mathbb{Q} \Rightarrow \exists a, b \in \mathbb{Z}$ st $k = a/b \wedge$ a, b do not have any common factors   rationality

③ $S \subseteq T \wedge T \subseteq S \Rightarrow S = T$ set theory

→ need to prove both!

thanks for coming! help me help you → https://www.tinyurl.com/aishani-sp21-fb