recap

- will usually be inducting on $\mathbb{N}$ in this class

General Induction Process

1. base case → $P(BC)$ pushing over first domino
2. inductive hypothesis → $P(k)$, $k \geq BC$ the k^{th} domino has fallen
3. inductive step → $P(k) \Rightarrow P(k+1)$ k^{th} domino falling makes $k+1^{th}$ fall

Strengthening Hypothesis (≠ Strong Induction!)

give proof more structure by proving pattern that is more specific than original claim.

try to seek out pattern by looking @ first few terms

False Proofs

ensure that no part of your inductive step requires k to be anything specific (other than higher than the base case)

- Same Color Horses proof : assumes $k \geq 3$, but $BC = 1$.

recap

Weak Induction vs. Strong™ Induction

Weak

1. base case → $P(BC)$
2. inductive hypothesis → $P(k)$, $k \geq BC$
3. inductive step → $P(k) \Rightarrow P(k+1)$

Strong

1. base case → $P(BC)$
2. inductive hypothesis → $P(i)$, $BC \leq i \leq k$
3. inductive step → $\bigwedge_{i=BC}^{k} P(i) \Rightarrow P(k+1)$

Now, suppose we have some

$$P'(k) := \bigwedge_{i=BC}^{k} P(i).$$

$$P'(k+1) = \bigwedge_{i=BC}^{k+1} P(i) = P'(k) \wedge P(k+1)$$

WTS: $P'(k) \Rightarrow P'(k+1)$

$\neg P'(k) \vee P'(k+1)$

$\neg P'(k) \vee (P'(k) \wedge P(k+1))$

TRUE $\cancel{(\neg P'(k) \vee P'(k))} \wedge (\neg P'(k) \vee P(k+1))$

which is really just showing $P'(k) \Rightarrow P(k+1)$

not technically in scope... but a way of knowing that strong induction to prove P is just weak induction to prove P'! ö

key takeaways

- Strong induction is just weak induction, "reformatted"
- inductive step: $\forall n \geq BC$, $P(n) \Rightarrow P(n+1)$
- Fibonacci Problems: when in doubt, try to leverage — $F(k) = F(k-1) + F(k-2)$
  $F(k) \geq F(k-1)$
  → usually need two base cases (why?)
- you're just showing $P(k+1)$ under the assumption that $P(k)$ is true
  → can often write out expression for $P(k+1)$ and group $P(k)$ together (common in algebraic proofs)

thanks for coming! help me help you → https://www.tinyurl.com/aishani-sp21-fb