recap Optimality

Intuition: "lowering standards only as much as they need to"

Proof: P&R is job optimal. assume some J with optimal C, J' w/ optimal C'

Suppose NOT job-optimal matching thru P&R so in matching T, J is paired w/ non optimal C^*

- we know J: ... $C > C^*$ ... (preference).
- on day K, J got rejected by C. suppose for simplicity, K is first day where a job gets rejected by its optimal candidate.
- on day K, C rejected J for J'. J' has not been rejected by its optimal cand. yet, so we know J': ... C > optimal cand (c^*) ... b/c J' proposed to C today.
- by definition of optimality, there is a matching S where (J, C), (J', C_2) where C_2 is just some other candidate that is a valid match for J'.
- on day K of making matching T: J' has not been rejected by C_2 (b/c C_2 is valid, so at least as bad as J's optimal C')
- we know J' ... $C > C_2$

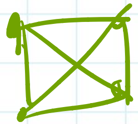
assuming T comes from P&R, S is just any other stable matching

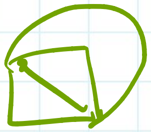
- C and J' are rogue couple in matching S = but we assumed R is stable

recap

$G = (V, E)$ mostly work with undirected graphs in 70.

Definitions





- vertices u, v adjacent / neighbors = share edge
- connected graph = $\exists$ path between any 2 vertices
- path $\subseteq$ walk (path cannot repeat vertices / edges, walk can)
  tour $\subseteq$ walk (tour needs to start & end in same place)
  cycle $\subseteq$ tour (cycle cannot repeat vertices <u>except</u> the start / end)
- planar: can be drawn on paper w/o crossing (ok to curve edges)
  nonplanar $\Longleftrightarrow$ (contains K_5 or $K_{3,3}$)
- bipartite: vertices can be split into 2 disjoint sets, edges go between sets only.
- Euler's formula: $v + f = e + 2$ (infinite face counts as 1)
- Coloring: soon!

recap Graph Induction

"Shrink down, grow back"

① base case (often $v=1$ or $e=1$)

② assume true for size n graph

③ start with size $n+1$ graph, remove vertex/edge, apply ②, then show everything holds when you add the vertex/edge back in.

What is wrong with the following "proof"?

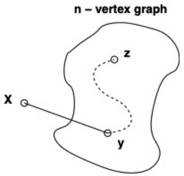
False Claim: If every vertex in an undirected graph has degree at least 1, then the graph is connected.

Proof: We use induction on the number of vertices $n \geq 1$.

Base case: There is only one graph with a single vertex and it has degree 0. Therefore, the base case is vacuously true, since the if-part is false.

Inductive hypothesis: Assume the claim is true for some $n \geq 1$.

Inductive step: We prove the claim is also true for $n+1$. Consider an undirected graph on n vertices in which every vertex has degree at least 1. By the inductive hypothesis, this graph is connected. Now add one more vertex x to obtain a graph on $(n+1)$ vertices, as shown below.

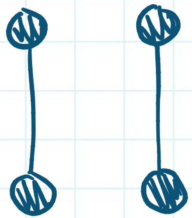


This proof only shows that all n+1-vtx graphs built up in this specific way satisfy the statement.

All that remains is to check that there is a path from x to every other vertex z. Since x has degree at least 1, there is an edge from x to some other vertex; call it y. Thus, we can obtain a path from x to z by adjoining the edge $\{x,y\}$ to the path from y to z. This proves the claim for $n+1$. * □

* assumes that all n+1-vtx graphs with property M can be built from a n-vtx graph with property M.

take $n=3$. M = all vtx degree >0



There's no way to build this from a 3-vtx graph that also satisfies M.

n+1 vertex graph (satisfies M)