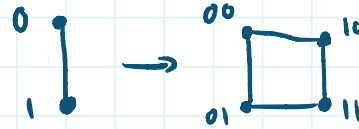# recap

Trees   all equivalent definitions

- connected and acyclic
- connected and $|E| = |V| - 1$
- maximally acyclic (adding any edge will create cycle)
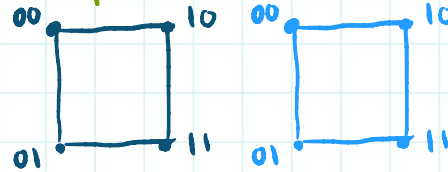- minimally connected (removing any edge disconnects graph)

Hypercube Properties:

- edges exist between vertices that differ on exactly one bit, eg $010 \text{——} 000$
- $n \cdot 2^{n-1}$ edges
- $E(n) = 2E(n-1) + 2^{n-1}$
- each vertex : degree n

Hypercubes

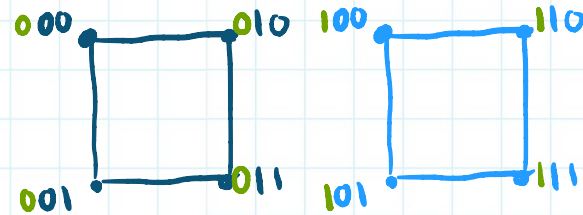n-dimensional hypercube $= 2^n$ vertices

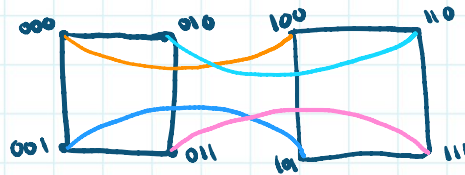how to go from $(n-1)$ to n dimension:

① duplicate $(n-1)$ cube

② add 0s on 1 side, 1s on the copy

③ add edges b/w vertices that are all same except 1st bit

(this is the same as the cube i promise)

we went from all length 2 bitstrings to all length 3 bitstrings!

- trees sparsely connected
- hypercubes more densely connected
- Complete (K_x) fully connected

key takeaways miscellaneous facts

- if two paths diverge then converge, the graph has a cycle
- bipartite $\equiv$ 2 vertex colorable
- bipartite $\equiv$ no odd cycles

- complete graph w/ n vertices has $\frac{n(n-1)}{2}$ edges