recap

functions

$f: X \to Y$ maps each $x \in X$ to some $y \in Y$

injective at most one x mapped to each y
↳ aka one-to-one, each x has unique image

surjective at least one x mapped to each y
↳ aka onto, each y has a pre-image

bijection™ exactly one x mapped to each y
↳ f is bijection $\iff$ f has inverse function

countable : be able to inject into naturals.

bijections are injective and surjective.

to prove $|X| = |Y|$ can show a bijection between them.

Proving 1:1 and onto for functions with some formula —

1. one-to-one : start with $f(x) = f(x')$ and show $x = x'$
2. onto : solve for x in terms of $f(x)$

counting infinite sets trickier!

0, 1, 2 ...

naturals

hard to count, so we compare to a set we know is countably infinite : $\mathbb{N}$:)

WTS bijection from naturals → [whatever we are trying to prove is countably inf]

recap

Countably Infinite

- $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{N} \times \mathbb{N}$
- union of countably many countable sets
- all finite-length bitstrings

Uncountably Infinite

- $\mathcal{P}(\mathbb{N})$ (power set of $\mathbb{N}$)
- infinite length bitstrings $\to \{0,1\}^\infty$
- $\mathbb{R}$ (reals)
- $(0,1)$

if $|A| \leq |B|$ and B is countable, so is A.
if $|A| \geq |B|$ and B is uncountable, so is A.
if $|A| = |B|$ either they're both countable or both uncountable.

counting sets of functions: really depends on individual problem but remember that for all $f: X \to Y$, each f is uniquely determined by ordered set of y vals that fixed x vals map to.

is Set S countable?

- maybe countable
  - come up with way to enumerate them
    Dis 6B q2B
  - can you enumerate it without skipping stuff? → establish bijection to $\mathbb{N}$
    - → injection + surjection
    - → injection from $S \to \mathbb{N}$ and from $\mathbb{N} \to S$
- maybe uncountable
  - establish bijection to uncountable set
  - Cantor's diagonalization

oh you love countable sets? name every number