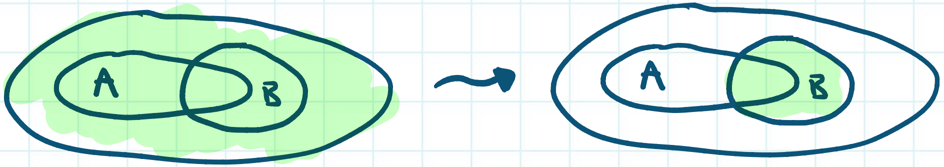
# recap

**conditional probability** = shrinking our sample space




$P[A|B] \cdot P[B] = P[A \cap B]$

$P[A|B]$ = "how much of B (given) $\underbrace{\text{also lies in A}}_{}$" → $A \cap B$

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

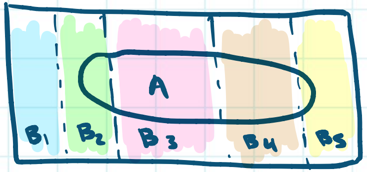
**bayes' rule**

$$P[A|B] = \frac{P[B|A]\,P[A]}{P[B]}$$

don't forget to use total prob rule in denominator

**total probability rule**

A = circle




$$P[A] = \sum_n P[A \cap B_n]$$

$$P[A] = \sum_n P[A|B_n] \cdot P[B_n]$$

" adding up all the subspaces of the sample space"

# recap

## independence

$\boxed{P[A \mid B] = P[A]}$

intuitively, knowing B happened did not give us any info about how likely A is.

$$\boxed{P[A \cap B] = P[A] \cdot P[B]}$$

* disjoint events are edit: **usually not** independent.

events $A_1, A_2 \ldots A_n$ are mutually independent if for all possible subsets of this set of events, $P[A_1 \cap \ldots \cap A_k] = P[A_1] \cdot P[A_2] \cdot \ldots P[A_k]$

pairwise independent if above is true for all subsets of size 2.

mutually independent $\Rightarrow$ pairwise independent (but not other way around)

## intersection of events

$$\boxed{\begin{aligned} P[A \cap B] &= P[A] \cdot P[B \mid A] \\ P[A \cap B \cap C] &= P[A] \cdot P[B \mid A] \cdot P[C \mid A \cap B] \end{aligned}}$$

## union of events

$$\boxed{\begin{aligned} P[A \cup B \cup C] = {} & P[A] + P[B] + P[C] \\ & - P[A \cap B] - P[B \cap C] - P[A \cap C] \\ & + P[A \cap B \cap C] \end{aligned}}$$

## union bound

$$\boxed{P\left[\bigcup_{i=1}^{n} A_i\right] \le \sum_{i=1}^{n} P[A_i]}$$

exactly equal if all events are disjoint