recap

Expectation

$$E[X] = \sum_{a \in \mathcal{A}} a \cdot P[X=a]$$

Important Facts

✓ $E[X+Y] = E[X] + E[Y]$

✓ $E[cX] = cE[X]$

✗ $E[XY] = E[X]E[Y]$

✗ $E[\frac{1}{X}] = \frac{1}{E[X]}$

these are not always true.

"sums, differences, and constant multiples of RVs"

How to use Linearity of Expectation

① goal: find $E[X]$

② figure out how X can be split up into $X_1, X_2 \ldots X_n$ st $X = X_1 + X_2 + \ldots + X_n$

- X_i should be indicator vars (Bernoulli)
- do not need to be independent
- consider "subcases"

then,

③ find $E[X_i]$

④ use linearity of expectation :)

recap

Remember this problem?

3 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let X denote the number of queens you draw.

(a) What is $\mathbb{P}(X=0)$, $\mathbb{P}(X=1)$, $\mathbb{P}(X=2)$ and $\mathbb{P}(X=3)$?

(b) What do your answers you computed in part a add up to?

(c) Compute $\mathbb{E}(X)$ from the ~~definition of expectation~~. linearity of expectation™

(d) Let X_i be an indicator random variable that equals 1 if the ith card a is queen and 0 otherwise. Are the X_i indicators independent?

why indicators are cool:

- don't need to be independent
- Bernoulli, so $E[I]$ is really easy to find ~

(c) X = # of queens drawn

$$X_i = \begin{cases} 1, & \text{card } i = \text{queen} \\ 0, & \text{otherwise} \end{cases}$$

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A of ◇, 5 of ♡

$$P[X_i] = \frac{1}{13}$$

$$X = X_1 + X_2 + X_3$$

$$E[X] = E[X_1] + E[X_2] + E[X_3] = \frac{3}{13}$$

recap Hypergeometric Distribution

like Binomial, measuring # of successes out of n trials
BUT without replacement

intuitively™, getting one success reduces probability of next success.

$$X \sim \text{Hypergeometric}(N, B, n)$$

"total things" — # of total possible success — total trials

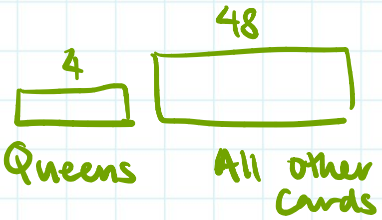
$$P[X = k] = \frac{\binom{B}{k}\binom{N-B}{n-k}}{\binom{N}{n}}$$

Cards:

X = # of Q when drawing 3 cards

$n = 3$

$N = 52$



pick k Qs out of 4 choices

pick rest of cards

$$\frac{\binom{B}{k}\binom{N-B}{n-k}}{\binom{N}{n}}$$