recap

Recall ... $\text{Var}(X) = E[X^2] - E[X]^2$

Covariance how two RVs are related $\quad \text{Cov}(X,Y) = E[XY] - E[X]E[Y]$

① $\text{Cov}(X,X) = \text{Var}(X)$

② X, Y independent $\Rightarrow \text{Cov}(X,Y) = 0$

converse is NOT true ... consider →

$X = \begin{cases} -1 & p = 1/2 \\ 1 & p = -1/2 \end{cases}, \quad Y = X^2$

$E[X] = 0$

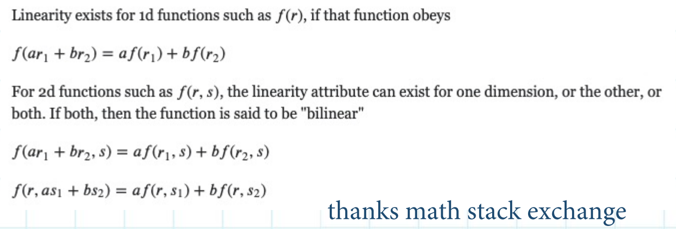
$E[Y] = 0$

$E[XY] = -1 \cdot 1 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{2}$

not indep. b/c Y is a function of X.

Covariance is bilinear, so

$$\text{Cov}\left(\sum_{i=1}^{n} a_i X_i, \sum_{j=1}^{m} b_i Y_i\right) = \sum_{i=1}^{n} \sum_{j=1}^{m} a_i b_j \text{Cov}(X_i, Y_j)$$



Linearity exists for 1d functions such as $f(r)$, if that function obeys

$$f(ar_1 + br_2) = af(r_1) + bf(r_2)$$

For 2d functions such as $f(r, s)$, the linearity attribute can exist for one dimension, or the other, or both. If both, then the function is said to be "bilinear"

$$f(ar_1 + br_2, s) = af(r_1, s) + bf(r_2, s)$$

$$f(r, as_1 + bs_2) = af(r, s_1) + bf(r, s_2)$$

thanks math stack exchange

↗ useful for covariance with indicator RVs! (q3 on this disc)

for all RVs, $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + \underbrace{2\,\text{Cov}(X,Y)}_{0,\text{ if independent}}$

Correlation

always between $[-1, 1]$

$$\text{Corr}(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma(X)\,\sigma(Y)}$$

σ = standard deviation

$= \sqrt{\text{variance}}$

recap Estimation

Problem: we want to estimate Y after observing X

$$\underset{\text{(observe)}}{X} \longrightarrow \underset{\underset{\text{inference function}}{\uparrow}}{g} \longrightarrow \hat{Y}$$

General Goal:

find g st we minimize the error between Y and $\hat{Y}$.

Note: usually minimizing square of the error is easier to do.

Linear Least Squares Estimate

- we know the joint distribution
- find $g(X) = a + bX$ that minimizes $E(|\underbrace{Y}_{\text{actual}} - \underbrace{g(X)}_{\text{estimate}}|^2)$

for now, it MUST be a linear function.

- $g(X)$, or $L[Y|X] = E[Y] + \dfrac{\text{Cov}(X,Y)}{\text{Var}(X)}(X - E[X])$

recap

Linear Regression

- don't know the exact joint distribution
- observe $(X_1, Y_1), (X_2, Y_2) \dots (X_k, Y_k)$
- find a, b st we minimize $\frac{1}{k}\sum_{i=1}^{k} |Y_i - a - bX_i|^2$
- as $k \to \infty$, this estimate converges to the LLSE

Minimum Mean Squares Estimate

- know the joint distribution
- Still trying to find $g(X)$ that minimizes $E(|\underbrace{Y}_{\text{actual}} - \underbrace{g(X)}_{\text{estimate}}|^2)$
- $g(X) = E[Y|X]$

  (will be a function of X)

Conditional Expectation — next time!

recap

Unbiased Estimator

estimate some quantity p with 95% confidence, allow 0.01 error

define RV $\hat{p}$ for our estimate.

$$\underbrace{P[|\hat{p} - E[\hat{p}]| \geq 0.01] \leq \frac{Var(\hat{p})}{0.01^2}}_{\text{by chebyshev's}} \leq 0.05$$

to use this, we want $E[\hat{p}] = p$ because what we REALLY want to compare is error b/w $\hat{p}$ and p.

with an unbiased estimator, $|\hat{p} - E[\hat{p}]| = |\hat{p} - p|$ yay!