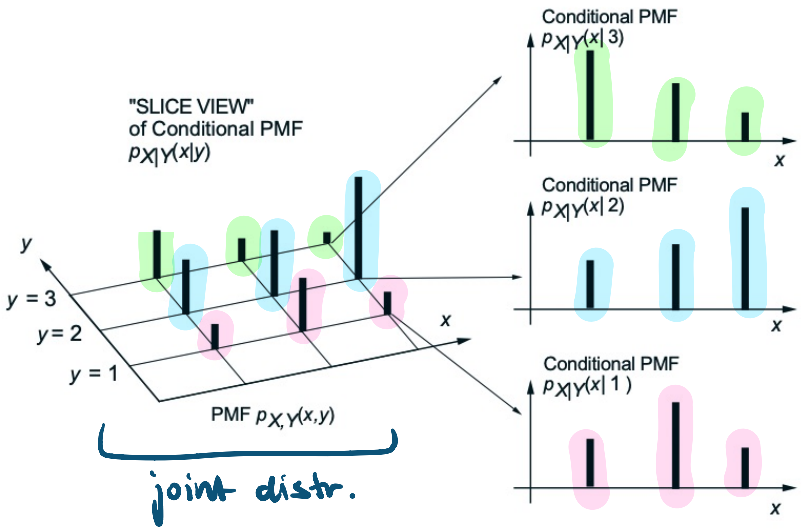
recap Conditional RVs

again, we use empirical knowledge of Y to estimate X → $X|Y$



$$P[X=k] = \sum_y P[X=k \mid Y=y] \cdot P[Y=y]$$

Now let's do conditional expectation.

$E[X \mid Y=y]$ will have a specific value, specifies which slice to look at. (if $y=2$, find expected value of blue distribution)

$E[X|Y]$ is a function of Y.

$= f(y)$ where $f = E[X \mid Y=y]$

we don't know what Y is, so what slice do we look at?

$E[X|Y]$ itself is an RV, that depends on value of Y (that depends on ω).

$$\underset{\text{outcome}}{\omega} \rightarrow Y(\omega) = y \rightarrow E[X \mid Y=y]$$

recap **Rules of Conditional Expectation**

Law of Total Expectation — $E[X] = E[E[X|Y]]$

Fact ———— $E[X|Y=y] = \sum_x x\, P[X=x, Y=y]$

Total Expectation Thm — $E[X] = \sum_y E[X|Y=y]\, P[Y=y]$

couldn't find in notes but it's used in one of the disc solutions.

Linearity — $E[a_1X_1 + a_2X_2 \mid Y] = a_1E[X_1|Y] + a_2E[X_2|Y]$

useful for breaking up indicators woohoo

If goal is to find $E[X]$ and distribution of X changes based on some factor, you should use conditional expectation.