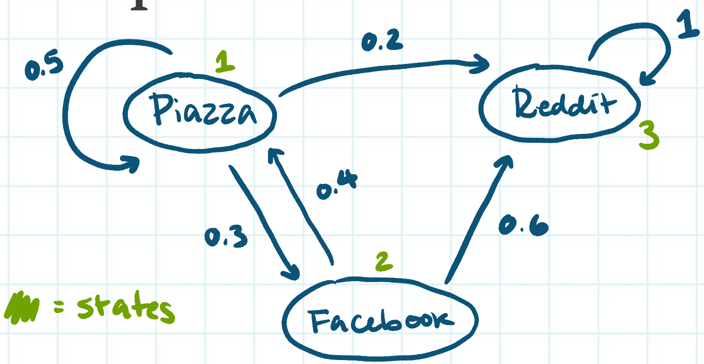
recap Markov Chains



= states

$\pi_0 = [\,0.8 \quad 0.1 \quad 0.1\,]$

$\pi_1 = \pi_0 P =$

$$[0.8 \quad 0.1 \quad 0.1] \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0 & 0.4 & 0.6 \\ 1 & 0 & 0 \end{bmatrix}$$

$= [\,0.5 \quad 0.28 \quad 0.22\,]$

State space $\mathcal{X} = \{1, 2, \dots k\}$

eg. {Piazza, FB, Reddit}

$$\begin{array}{ccc} \text{Piazza} & \text{FB} & \text{Reddit} \\ 1 & 2 & 3 \end{array}$$

transition probability matrix

eg. $P \rightarrow \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0 & 0.4 & 0.6 \\ 1 & 0 & 0 \end{bmatrix}$

Rows will sum up to 1 ... "we have to go somewhere"

probabilities $\pi_i(k) = P[$we're in state k at timestep $i]$

$P(i,j) =$

$P(\text{from}, \text{to})$

$\pi_i = [\,\pi_i(1) \quad \pi_i(2) \quad \dots \quad \pi_i(k)\,]$

sum of these elements should be 1.

$\pi_{i+1} = \pi_i P$

$\pi_n = \pi_0 P^n$

current state X_i = RV representing state @ timestep i

$P[X_i = b \mid X_{i-1} = a] = P(a,b)$

recap more definitions wow

invariant distribution $\pi = \pi P$

- to solve for invariant, use $\pi = \pi P$ and $\sum_i \pi(i) = 1$
- if π_0 is the invariant, then $\pi_n = \pi_0$ for all n.

Markov property "future depends on only the present, not the past"

$$P[X_n = i_n \mid \underbrace{X_{n-1} = i_{n-1}, X_{n-2} = i_{n-2} \ldots}_{\text{tracking every state we've been in since the beginning}}] = P[X_n = i_n \mid \underbrace{X_{n-1} = i_{n-1}}_{\text{tracking just the last state}}]$$

reducibility + periodicity — next time!

recap

Hitting Time / A Before B

want to consider all states

① find "fixed" probabilies for some states

- usually one will be our goal
- some might represent that "we will never get to our goal"

② express α(rest of the states) in terms of ①

for each state, think about where we can go next, and with what probability

Tip

whether you add 1 in your equations or not depends on if advancing a step costs you something (time, coin flips, etc) like when you're calculating the expected number of something.