recap

1 Markov Chain Terminology

In this question, we will walk you through terms related to Markov chains.

1. (Irreducibility) A Markov chain is irreducible if, starting from any state i, the chain can transition to any other state j, possibly in multiple steps.
2. (Periodicity) $d(i) := \gcd\{n > 0 \mid P^n(i,i) = \mathbb{P}[X_n = i \mid X_0 = i] > 0\}, i \in \mathscr{X}$. If $d(i) = 1\ \forall i \in \mathscr{X}$, then the Markov chain is aperiodic; otherwise it is periodic.
3. (Matrix Representation) Define the transition probability matrix P by filling entry (i, j) with probability $P(i, j)$.
4. (Invariance) A distribution π is invariant for the transition probability matrix P if it satisfies the following balance equations: $\pi = \pi P$.

] Can we get to any state from any other state?

] How long can we take to start and end @ state i? (sort of)

n = any possible # of steps it can take to get from state i back to i.

aperiodic if $d(i) = 1$ FOR ALL STATES

- every finite state Markov Chain has at least one invariant distribution
- if irreducible, invariant distribution is unique
- if irreducible + aperiodic, we always converge to the invariant distribution (ie $\pi_n \approx \pi$ as $n \to \infty$)
  - if periodic, initial distribution might not converge to invariant
  - long term fraction of time spent in each state follows π if irreducible.

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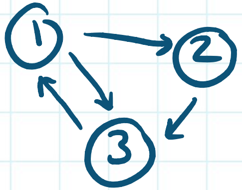
irreducible + d(i) of one node = 1

=> d(i) of all nodes = 1 => aperiodic

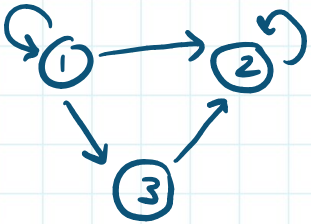
intuition (not proof)

imagine if one node I has self loop.
for any other node k, we can walk
$k \to \dots \to I \to \dots \to k$ as well as
$k \to \dots \to I \to I \dots \to k$. gcd = 1

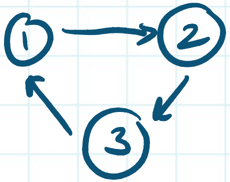
Irreducible



Reducible

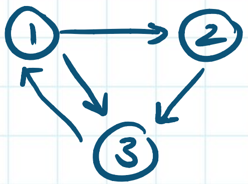


Periodic



$1 \to 2 \to 3 \to 1$

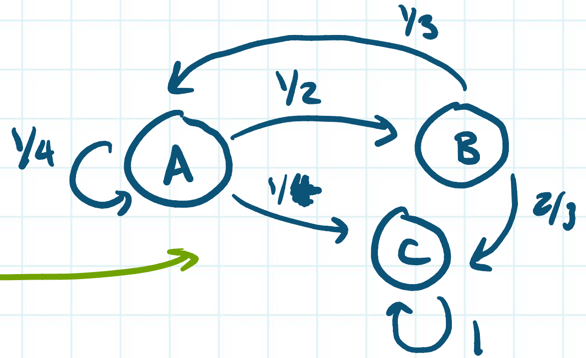
Aperiodic



d(i) is for a specific state i.

recap Hitting Time / Probability

can kind of think of it like recursion



Find expected # of steps to get from A → C.

Find n!

Recursion

① base case

$n! = 1$ if $n \leq 1$

② recursive step / leap of faith

$n! = n \cdot \underbrace{(n-1)!}_{\text{already have this}}$

Hitting Time

① base case

when have I reached my goal, or made it impossible to reach?

$\beta(C) = 0$

already @ C, so # steps to C = 0

② express in terms of expected value of future states.

CAUTION: hitting time isn't EXACTLY recursion, this is just to help understand. with recursion, we'd have issues if you needed to know n! to find (n-1)! but here all we want to do is set up a solvable system of equations.

no +1 if doing probability usually. but same general approach.

$\beta(B) = 1 + \frac{2}{3}\beta(C) + \frac{1}{3}\beta(A)$

we aren't at C, so it will take at least 1 step to get to C.

after taking that step, we're either at C or A.