recap Continuous Probability

$P[X=x]=0$, consider $P(x \le X \le x+dx)$ instead

always consider over an interval.

Probability Density Function $f_X(x)$

relevant discrete comparison... lot of properties are very similar

$f_X(x) \ge 0$ $\quad\quad$ $P[X=x] \ge 0$

$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ $\quad\quad$ $\sum_x P[X=x] = 1$

$f_X(x)$ evaluated @ 1 value isn't really meaningful because $P[X=x]=0$

but we can integrate the pdf to get the relative likelihood of X falling in some range.

Cumulative Distribution Function $F_X(x)$

$$F_X(x) = P[X \le x] = \int_{-\infty}^{x} f_X(x)\,dx$$

pdf is the derivative of the CDF.

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Characterizing the Distribution(s)

Expectation

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

Discrete parallel:

$$E[X] = \sum_x x\, P[X = x]$$

Variance

$$Var(X) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \left(\int_{-\infty}^{\infty} x f(x)\, dx \right)^2$$

$$Var(X) = E[X^2] - E[X]^2$$

Joint Density Functions

$f(x,y) \geq 0 \quad \forall x, y \in \mathbb{R}$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$$

$$P[a \leq X \leq b,\ c \leq Y \leq d] = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

X, Y independent if

$$f(x,y) = f_X(x)\, f_Y(y)$$

$\forall x, y \in \mathbb{R}$. also if

$$f_{(Y|X)}(y|x) = f_Y(y)$$

get marginal distr. for X by integrating joint density wrt. Y.

recap Example Distributions

standard normal: $\mu = 0$, $\sigma = 1$

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Uniform (a, b)

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{else} \end{cases}$$

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$$

$$E[X] = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

Exponential (λ)

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{else} \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$

Normal (μ, σ^2)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

$$F(x) = \phi\left(\frac{x-\mu}{\sigma}\right)$$

standard normal cdf.

$$E[X] = \mu$$

$$Var(X) = \sigma^2$$

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Steps For Solving Problems Involving 2 RVs

Disclaimer: I did not come up with this, just learned this technique when I took the class :)

general tip: consider how you would do it if it were a discrete problem, then think about what changes to make (ie sums → integrals, $P[X=x] \rightarrow f_X(x)\,dx$, etc)

① plot your RVs on a 2D plane and shade the feasible region

② find the bounds of the feasible region, eg $\int \int \boxed{} \, d\square \, d\square$

- our domain of integration usually won't be too crazy, but do some practice on this

③ find / determine pdfs of your RVs

- sometimes CDF is easier to find, so get CDF and derive.

④ if both are uniform, find area of region (like just $\frac{1}{2} b \cdot h$ kinda stuff)

else, integrate over the joint density function.